Bogoliubov Condensation of Gluons and Spontaneous Gauge Symmetry Breaking in QCD

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MPG–VT–UR 60/95
December 1995

The problem of the gluonic quasiparticle excitations in QCD is considered under the aspect of the condensation of gluon pairs in the "squeezed" vacuum. The present approach is a field theoretical generalization of the Bogoliubov model which successfully reproduced the Landau spectrum in the microscopic theory of superfluidity. We construct a gauge invariant QCD Hamiltonian by formally solving the Gauss equation such that the physical variables are separated by a non-Abelian projection operator, instead of fixing a gauge. By using Dirac quantization we show that the Bogoliubov condensation of gluon pairs destroys this projection operator, and the spontaneous appearance of a gluon mass is accompanied by a longitudinal component for the gluon field in correspondence with the relativistic covariance. Gauge symmetry is broken spontaneously since the gauge invariance of the Hamiltonian is not shared by the vacuum. The squeezed vacuum in the present model is characterized by one free parameter related to the contraction of a pair of zero momentum gluon fields which is fixed from the difference of the η' and the η - meson masses ($U(1)$-problem) and results in a value for the gluon condensate which is in good agreement with the value obtained by Shifman, Vainshtein and Zakharov.

PACS number(s): 11.15.Ex, 12.38.Aw, 14.70.Dj

I. INTRODUCTION

The problem of the gluon vacuum in QCD has a long history [1–6]. It is well known that at low energies perturbation theory does not apply since the coupling constant becomes large and the perturbative vacuum becomes unstable due to gluon self-interactions [1]. These self-interactions can lead to a 'reconstruction' of the vacuum and to the appearance of a condensate [7,8]. The presence of the condensate is important for the physical properties of the low energy sector of QCD. Of particular interest is the modification of the quasiparticle spectrum and the occurence of massive collective excitations to be considered in this work.

In the literature two different kinds of such condensates have been considered, the *coherent* and the *squeezed* one. In the coherent condensate gluon field excitations are found by a *transitive* transformation, i.e. shifting the fields to the solution of the classical equations [1–3] (e.g. instantons [4–6]). It is characterized by the condensation of single gluons and thus by a nonvanishing vacuum expectation value of the gluon field A:

$$< A > \neq 0 \ . \tag{1}$$

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In the squeezed condensate on the other hand the gluon states are constructed by a *multiplicative* Bogoliubov transformation of the gluon fields [9–13]. The squeezed vacuum is characterized by the condensation of colourless, scalar gluon pairs and could thus be realized through

$$< A > \; = \; 0 \; , \;\; < A^2 > \; \neq 0 \; . \tag{2}$$

There has been a lot of activity to construct a stable coherent vacuum in the gluon sector of QCD [1–3]. The problem in this case is that there are no stable quasiclassical solutions to the Yang-Mills equations in Minkowski space [2,3]. In recent years the squeezed condensate (called here *Bogoliubov condensate*) has become a topic of great interest, see e.g. [9–13]. Its investigation for non-Abelian fields faces the following problems:

1. One has to find the adequate degrees of freedom to construct the gluon condensation in a squeezed vacuum.

2. Since the squeezed vacuum is most naturally described in the Hamiltonian formalism one has to extract the gauge invariant oscillator - like field variables from the non-Abelian QCD action. Recall that by using different gauges for instance Biro [10,11] and Mishra [12,13] get different results for the condensate.

3. The condensate leads to spontaneous gauge symmetry breaking with the appearance of massive gluons. This has to be accompanied by the corresponding generation of the longitudinal components for the massive gluon fields. Recent papers on "squeezed" gluon states [10–13] indeed obtain a constituent gluon mass. However, since they fix the gauge, their gluon fields do not have a longitudinal component such that they are unable to provide the number of degrees of freedom required for the description of a massive gluon vector field [14].

With respect to the last topic, we mention that mass generation for gauge fields via spontaneous gauge symmetry breaking is a general problem familiar also from other gauge field theories. In the unified theory of electroweak interactions a consistent description of massive vector fields can be given due to the presence of a scalar Higgs field in the Lagrangian which generates the longitudinal component of the massive gauge bosons W and Z. The very interesting alternative, that the gauge bosons obtain their mass by spontaneous gauge symmetry due to radiative corrections, without introduction of an external Higgs field, has been proposed by Coleman and Weinberg [15]. This possibility is of great importance since for SU(N) gauge theories the introduction of an external Higgs field does not lead to a mass term for the gluons, as was shown by Georgi and Glashow [16]. The concept of spontaneous gauge symmetry breaking by radiative corrections is therefore very attractive for the case of QCD and has been followed up to now [10–13].

In the present work we study the possibility of gluon condensation in a squeezed vacuum in view of the Bogoliubov model [7] of the weakly nonideal Bose gas. In particular we investigate the influence of a squeezed condensate on the gluon quasiparticle spectrum in the low-energy region of QCD. Note that the Bogoliubov theory was the first to explain the experimentally observable spectrum of collective excitations of superfluid ^{4}He. This spectrum cannot be obtained by resummations of the conventional perturbation theory series. As Bogoliubov has shown, the collective exitations are determined by the "condensate" of particles with zero momentum and finite density. A first connection between Bogoliubov condensation and the squeezed vacuum in field theory (massless $\lambda\phi^4$) was made by Castorina and Consoli [17], see also [18]. An application of the concept of the Bogoliubov model to QCD, however, has to our knowledge not been carried out by now.

In a first attempt to generalize the Bogoliubov model to QCD we use the infrared singularity of massless theories to squeeze the zero momentum mode which leads to massive gluonic quasiparticles in the nonzero momentum sector. The free squeezing parameter is fixed from the $\eta - \eta'$ mass difference. Our coresponding value for the gluon condensate is in reasonable agreement with that obtained by Shifman, Vainshtein and Zakharov [5] which supports our semi-phenomenological approach.

Concerning the above mentioned problems (2) and (3), we try to solve the problem of the appearance of a constituent gluon mass using a gauge invariant scheme for the elimination of the unphysical components of the gluon vector field [19–23] which does not require the gauge-fixing as initial supposition. This scheme is based on the construction of projection operators by formally solving the Gauss law constraint. We show that these projectors are destroyed by the interaction of gluons with the squeezed vacuum. As result a constituent gluon mass appears together with the necessary longitudinal components. This is the central result of our paper and is quite in analogy to spontaneous chiral symmetry breaking in the quark sector [24–26]. There, the appearance of constituent quark masses due to the interaction of quarks with the squeezed vacuum is accompanied by the destruction of the chiral projection operator which leads to the necessary increase of the number of spinor field components from two (Weyl spinors) to four (Dirac spinors).

The paper is organized as follows: In Section II, the Bogoliubov model of a weakly nonideal Bose gas is generalized to field theory. We give a field theoretical description of the condensation phenomenon by the use of the squeezed

vacuum and discuss the conventional local $\lambda\phi^4$ theory. The relation to the Bogoliubov model for the weakly nonideal Bose gas is given in Appendix A. In Section III the homogeneous colourless Bogoliubov condensate of gluons is introduced in QCD where the unphysical degrees of freedom are eliminated by applying projection operators instead of fixing a gauge. We also discuss spontaneous gauge symmetry breaking and the corresponding occurrence of a massive gluon quasiparticle spectrum. In Section IV the squeezing parameter is fixed ¿from the $\eta' - \eta$ mass difference. In Section V we present the conclusions.

II. BOGOLIUBOV CONDENSATION IN QUANTUM FIELD THEORY

In order to introduce some notations and methods needed for our investigation of the squeezed condensate in the rather complicated QCD, we first consider massless $\lambda\phi^4$ theory with the Hamiltonian

$$H = \int d^3x[\pi(x)^2 + (\partial_i\varphi(x))^2 + \frac{\lambda}{4!}\varphi^4(x)] \tag{3}$$

as the simplest example of an interacting bosonic theory which is renormalizable. The theory is quantized by turning the classical fields $\varphi(\mathbf{x},t), \pi(\mathbf{x},t)$ to Schrödinger operators $\varphi(\mathbf{x}), \pi(\mathbf{x})$ and imposing the canonical commutation relations

$$[\pi(\mathbf{x}), \varphi(\mathbf{x}')] = -i\delta(\mathbf{x}-\mathbf{x}') \ . \tag{4}$$

In the momentum representation defined by

$$\varphi_p = \frac{1}{\sqrt{V}}\int d^3x \mathrm{e}^{i\mathbf{px}}\varphi(\mathbf{x}), \qquad \pi_p = \frac{1}{\sqrt{V}}\int d^3x \mathrm{e}^{i\mathbf{px}}\pi(\mathbf{x}) \ , \tag{5}$$

the Hamilton operator is

$$: H[\varphi,\pi] := \frac{1}{2}\sum_p [: \pi_p\pi_{-p} : + p^2 : \varphi_p\varphi_{-p} :] + \frac{\lambda}{4!V}\sum_{p_1p_2p_3p_4}\delta_{p_1+p_2+p_3+p_4,0} : \varphi_{p_1}\varphi_{p_2}\varphi_{p_3}\varphi_{p_4} :, \tag{6}$$

with the commutation relations

$$[\pi_p, \varphi_{p'}] = -i\delta_{p,-p'} \ , \quad [\varphi_p, \varphi_{p'}] = [\pi_p, \pi_{p'}] = 0 \ . \tag{7}$$

In (6) we have introduced normal ordering with respect to the creation and annihilation operators a_p, a_p^+ defined according to

$$\varphi_p = \sqrt{\frac{1}{2\tilde{\omega}(p)}}(a_p + a_{-p}^+) \ , \qquad \pi_p = i\sqrt{\frac{\tilde{\omega}(p)}{2}}(-a_p + a_{-p}^+) \ , \tag{8}$$

with an arbitrary function $\tilde{\omega}(p)$. The operators a_p, a_p^+ satisfy the commutation relations

$$[a_p, a_{p'}^+] = \delta_{p,p'} \ , \quad [a_p, a_{p'}] = [a_p^+, a_{p'}^+] = 0 \ . \tag{9}$$

The corresponding vacuum $|0>$ is defined by $a_p|0>=0$, and the Fock space is given as

$$\{\,|\,\Phi>\} = \,|\,0>; \quad a_p^+|0>=|\,p>, \ldots \ . \tag{10}$$

We note that the special choice $\tilde{\omega}(p) = |p|$ diagonalizes the free (λ independent) part of the Hamiltonian. For this case, however, a_0 and a_0^+ are not defined which corresponds to the well-known infrared singularity of massless theories. Generalizing the Bogoliubov model to field theory we should use the infrared singularity of massless theories to squeeze the zero mode by populating it macroscopically with massless particles. Then, we diagonalize the nonzero mode single particle part of the resulting squeezed Hamiltonian by changing from particles to quasiparticles whose dispersion relation $\tilde{\omega}(p)$ is finally determined selfconsistently. For the simple case of $\lambda\phi^4$ this has been carried out in detail [18]. Similar to the Bogoliubov model this leads to renormalization of the bare parameters like the coupling constant λ.

For the time being we leave $\tilde{\omega}(p)$ open and suppose that the vacuum of the theory (6) contains a large number of quasiparticles with zero momentum ($p = 0$). We construct this vacuum using the unitary squeezing operator

$$U_B(\varphi_0, \pi_0) = \exp\left(i\frac{f_0}{2}(\pi_0\varphi_0 + \varphi_0\pi_0)\right) \ , \tag{11}$$

where f_0 is a very large parameter to be fixed later. The operator U_B transforms the Fock space of states to the Bogoliubov space of states

$$|\,\Phi_B >\equiv U_B^{-1}\,|\,\Phi > \ . \tag{12}$$

In quantum optics these states are called 'squeezed states', see e.g. [27].

Applying the unitary transformation (11), we can define the new field operator φ_0^B and its momentum π_0^B by means of

$$\begin{aligned} \varphi_0^B &= U_B^{-1}\varphi_0 U_B = \mathrm{e}^{-f_0}\varphi_0 \ , \\ \pi_0^B &= U_B^{-1}\pi_0 U_B = \mathrm{e}^{f_0}\pi_0 \ , \end{aligned} \tag{13}$$

which satisfy the same algebra of commutation relations as the initial ones (7).

We shall now carry out the squeezing of the zero mode part of the Hamiltonian by applying a Wick reordering procedure to the Bogoliubov vacuum $|0_B>$. Note that under the squeezing transformation (13) the contraction of a pair of field operators is left invariant,

$$C =<0\,|\,\varphi_0\varphi_0\,|\,0>=<0_B\,|\,\varphi_0^B\varphi_0^B\,|\,0_B> \ . \tag{14}$$

The normal ordering of the Bogoliubov fields φ_B with respect to the Bogoliubov vacuum (which is denoted as $::\varphi_0^B\varphi_0^B::$) has the same form as the normal ordering of the original fields with respect to the Fock vacuum (10)

$$C = \varphi_0\varphi_0 - :\varphi_0\varphi_0: = \varphi_0^B\varphi_0^B - ::\varphi_0^B\varphi_0^B:: \ . \tag{15}$$

To reorder the Hamiltonian (6) with respect to the new vacuum $|0_B>$ we use Eqs.(13) and (15). Reordering of the quadratic term gives

$$:\varphi_0\varphi_0: = ::\varphi_0^B\varphi_0^B:: \ \mathrm{e}^{2f_0} \ + \ \tilde{C} \ , \tag{16}$$

where

$$\tilde{C} = C\left(\mathrm{e}^{2f_0} - 1\right) \ . \tag{17}$$

Analogously we have

$$:\pi_0\pi_0: = ::\pi_0^B\pi_0^B:: \ \mathrm{e}^{-2f_0} \ + \ C^\pi\left(\mathrm{e}^{-2f_0} - 1\right), \tag{18}$$

with $C^\pi =$ Reordering of the quartic term gives

$$\begin{aligned} :\varphi_0\varphi_0\varphi_0\varphi_0: \ = \ &::\varphi_0^B\varphi_0^B\varphi_0^B\varphi_0^B::\mathrm{e}^{4f_0} + ::\varphi_0^B\varphi_0^B::\mathrm{e}^{2f_0}\tilde{C} + (5 \text{ permutations}) \\ &+\tilde{C}^2 + (2 \text{ permutations}) \ . \end{aligned} \tag{19}$$

For applications in QCD we quote here also the general Wick reordering result for any polynomial $F(\varphi_0)$

$$:F(\varphi_0): = \exp\{\frac{1}{2}\tilde{C}\frac{d^2}{db^2}\} :: F(\varphi_0^B\mathrm{e}^{f_0} + b) :: \Big|_{b=0} \ . \tag{20}$$

As result of the reordering of $H = H_0 + H'$, where H' is the Hamiltonian of nonzero momentum excitations ($p \neq 0$), we obtain

$$\begin{aligned} :H: \ &= ::H_0(\phi_0):: + :H': \ , \\ :H'[\varphi,\pi]: \ &= E_0 + :H^{(2)}[\varphi,\pi]: + :H^{(4)}[\varphi]: \ , \end{aligned} \tag{21}$$

where

$$E_0 = 3\frac{\lambda}{4!V}\tilde{C}^2 + C^\pi \tag{22}$$

$$: H^{(2)}[\varphi, \pi] : \; = \frac{1}{2}\sum_{p\neq 0}\left\{ : \pi_p \pi_{-p} : + \left[p^2 + \frac{\lambda}{2V}\tilde{C}_{00}\right] : \varphi_p \varphi_{-p} : \right\} \ , \tag{23}$$

$$: H^{(4)}[\varphi] : \; = \frac{\lambda}{4!V}\sum_{p_1,p_2,p_3,p_4\neq 0} \delta_{p_1+p_2+p_3+p_4,0} : \varphi_{p_1}\varphi_{p_2}\varphi_{p_3}\varphi_{p_4} : \ . \tag{24}$$

The zero momentum operator :: $H_0(\phi_0)$ containing terms proportional to :: $\varphi_0^B \varphi_0^B$ :: and :: $\varphi_0^B \varphi_0^B \varphi_0^B \varphi_0^B$ :: describes excitations of the condensate and has not been written explicitely here. Note that for very large f_0 the second term in the expression (22) for E_0 is much smaller than the first one (cf. (17)) and can be neglected. Hence we find a condensate energy density

$$\epsilon_0 \equiv \frac{E_0}{V} = \frac{\lambda}{8}\left(\frac{\tilde{C}}{V}\right)^2 \tag{25}$$

and a bosonic quasiparticle mass m_B,

$$m_B^2 \equiv \lambda \frac{\tilde{C}}{2V} \ , \tag{26}$$

appears. Diagonality of the one-particle part : $H^{(2)}$: of the reordered Hamiltonian (21) demands that we put the quasiparticle energy $\tilde{\omega}(p)$ in Eq. (8) to

$$\tilde{\omega}(p) = \sqrt{p^2 + m_B^2} \ . \tag{27}$$

The effective Hamiltonian depends (through ϵ_0 and m_B) on the free parameter $\tilde{C}$.

Using Eq. (26) we can eliminate the parameter $\tilde{C}$ ¿from the expression for the condensate energy density ϵ_0 in Eq. (25) to obtain

$$\epsilon_0 = \frac{m_B^4}{2\lambda} \ , \tag{28}$$

which has the same nonanalytic dependence on the coupling constant as that of the Higgs mechanism of mass generation. Obviously, the Bogoliubov mechanism of spontaneous mass generation presented above differs from the Higgs mechanism by the representations of the vacuum and the interaction of the quasiparticles. The Higgs mechanism corresponds to the coherent vacuum representation, while the Bogoliubov one - to the squeezed vacuum, see Appendix A. The introduction of the Bogoliubov condensate is related to the Wick reordering procedure with respect to a new Fock space.

III. BOGOLIUBOV CONDENSATE IN QCD

After the introductory generalization of Bogoliubov condensation for superfluid ^{4}He to massless $\lambda\phi^4$ theory above it is attractive to suppose that also the gluon vacuum of QCD can be considered as a homogeneous colourless condensate of gluon pairs. First steps in this direction where undertaken in Celenza and Shakin [9]. The corresponding treatment in QCD is far more complicated than in $\lambda\phi^4$ due to the fact that QCD is a gauge theory with unphysical degrees of freedom in the Lagrangian which have to be eliminated before quantization. According to Dirac [19], only the spatial components of the gauge fields are dynamical and have to be quantized. The time components obey constraint equations (Gauss laws) and have to be eliminated.

As in the simpler case of massless $\lambda\phi^4$ theory the squeezed condensate is described by the procedure of Wick reordering with a free parameter $\tilde{C}$ and leads both to a vacuum energy and to a mass term for the field. In QCD, however, the presence of a condensate and the corresponding generation of a mass term for the gauge field leads to spontaneous gauge symmetry breaking, since the gauge invariance of the Hamiltonian is not shared by the vacuum. The problem of the appearance of a constituent gluon mass und the corresponding resurrection of the longitudinal component of the massive quasigluons is solved by using a projection scheme for the elimination of the unphysical components of the gluon vector field [19–23] instead of gauge-fixing. The projectors are obtained by formally solving

the Gauss law constraint and appear in the kinetic energy term of a reduced gauge invariant QCD Hamiltonian. We show that these projectors are destroyed by the interaction of gluons with the squeezed vacuum so that the constituent gluon mass appears together with the necessary longitudinal components. The presence of a squeezed condensate leads to spontaneous gauge symmetry breaking: The gauge invariance of the QCD Hamiltonian is not shared by the vacuum.

Finally we fix the free parameter $\tilde{C}$ of our squeezed vacuum by estimating a value for the quasigluon mass from the $\eta' - \eta$ mass difference and comparing the corresponding condensate energy density to the well known value obtained by Shifman et al. [5]. We shall see that they are in good agreement.

A. QCD Hamiltonian and Gauss law

We start from the QCD Lagrangian

$$\mathcal{L}(A) = -\frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} \ , \tag{29}$$

where $F^a_{\mu\nu}$ is the field strength tensor

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu \ . \tag{30}$$

In the following, we use the notation

$$A_\mu = g \frac{A^a_\mu \lambda^a}{2i} \ , \tag{31}$$

where g is the coupling constant. Due to the gauge invariance

$$A^v_\mu = v(A_\mu + \partial_\mu) v^{-1} \ , \tag{32}$$

$$\mathcal{L}(A^v) = \mathcal{L}(A) \ , \tag{33}$$

this classical Lagrangian contains only $3(N_c^2 - 1)$ degrees of freedom instead of the $4(N_c^2 - 1)$ components of the A field (N_c is the number of colours). For the construction of the Hamiltonian one usually introduces chromoelectric and chromomagnetic fields

$$E^a_i = \dot{A}^a_i - D^{ab}_i(\mathbf{A}) A^b_0 \ , \tag{34}$$

$$B^a_i(\mathbf{A}) = \frac{1}{2} \, \epsilon_{ijk} F^a_{jk} = \epsilon_{ijk} D^{ab}_j(\mathbf{A}) A^b_k \ , \tag{35}$$

with $\dot{A}_i = \partial_0 A_i$ and the covariant derivative

$$D^{ab}_i(\mathbf{A}) = \delta^{ab} \partial_i + g \ f^{acb} A^c_i \ . \tag{36}$$

The magnetic field satisfies the Bianchi identity

$$D^{ab}_i(\mathbf{A}) B^b_i(\mathbf{A}) \equiv 0 \ , \tag{37}$$

which can be interpreted as a generalized transversality of the magnetic field.

In order to construct the Hamiltonian, we have to find the canonical momenta. We see that the Lagrangian (29) does not contain time derivatives of the zero components of the gluon fields. The corresponding Euler-Lagrange equations are therefore constraints (the Gauss laws):

$$D^2_{ab}(\mathbf{A}) A^b_0 = D^{ab}_i(\mathbf{A}) \dot{A}^b_i \ , \tag{38}$$

where $D^2_{ab} = D^{ac}_i D^{cb}_i$. In terms of the electric field the Gauss laws (38) read

$$G^a(\mathbf{A}, \mathbf{E}) \equiv D^{ab}_i(\mathbf{A}) E^b_i = 0 \ . \tag{39}$$

The canonical momenta to the spatial fields A^a_i are the electric fields:

$$\frac{\delta \mathcal{L}}{\delta \dot{A}^a_i} = E^a_i \ , \quad i = 1, 2, 3 \ . \tag{40}$$

The Hamiltonian can now be written as

$$H(\mathbf{A}, \mathbf{E}) = \int d^3x \frac{1}{2} \left[E^{a\,2}_i + B^{a\,2}_i \right] \ . \tag{41}$$

In the classical theory we thus have the Hamiltonian (41) together with the Gauss constraint (39).

B. Quantization

In order to quantize the theory one could write the Hamiltonian completely in terms of gauge invariant variables and their canonical conjugate momenta and then impose the canonical commutation relations only on these gauge invariant variables. Different to the case of QED, this leads to inconsistencies in QCD, as shown in detail in Appendix B.

A more successful alternative way is Dirac quantization [19], where one imposes the canonical commutation relations on the original A_i and E_i:

$$[E_i^a(\mathbf{x}), A_j^b(\mathbf{x}')] = i\delta^{ab}\delta_{ij}\delta(\mathbf{x}-\mathbf{x}') \ . \tag{42}$$

Both the Hamiltonian H in (41) and the Gauss function G^a in (39) then become operators satisfying

$$[H, G^a(\mathbf{x})] = 0 \ , \tag{43}$$

$$[G^a(\mathbf{x}), G^b(\mathbf{x}')] = if^{abc}G^c(\mathbf{x})\delta(\mathbf{x}-\mathbf{x}'). \tag{44}$$

Since G^a can be interpreted as the infinitesimal generator for gauge transformations these two commutation relations express the gauge invariance of the Hamiltonian (under small gauge transformations). Note that the Hamilton operator obtained from the classical Hamiltonian (41) with the cartesian fields E_i and A_i has the correct operator ordering [28]. As pointed out by Jackiw [29], the Gauss law $G^a = 0$ cannot be taken as an operator equation since it would lead to inconsistency with the Dirac commutation relations (42). Jackiw then suggested that the Gauss law should be implemented by demanding that a physical state satisfies the Schrödinger equation (with energy eigenvalue $\mathcal{E}$) and is annihilated by the Gauss law operator

$$H(\mathbf{A}, \mathbf{E})|\Phi> \ = \mathcal{E}|\Phi> \ , \tag{45}$$

$$G^a(\mathbf{A}, \mathbf{E})|\Phi> \ = 0 \ . \tag{46}$$

The second equation is the condition of gauge invariance of the physical states. The Gauss law constraint (46) can then at least in principle be implemented by use of unitary transformations [30] and is still under lively discussion [31]. The resulting kinetic term in the Hamiltonian is very complicated.

The requirement of gauge invariance of the physical states expressed by (46), however, is too restrictive. It does not allow for the possibility of spontaneous breaking of the gauge symmetry in analogy to spontaneous symmetry breaking in the Higgs model and spontaneous chiral symmetry breaking. If the gauge symmetry is broken spontaneously, the gauge invariance of the Hamiltonian is not shared by the vacuum.

We can arrive at a gauge invariant Hamiltonian without demanding gauge invariance of the physical states, especially of the vacuum

$$G^a(\mathbf{E}, \mathbf{A})|0_B> \neq 0 \ , \tag{47}$$

by starting from a gauge invariant reduced classical Hamiltonian. This is achieved by a projection method described in the following.

Using the formal solution of the Gauss equations (38)

$$A_0^a[\mathbf{A}] = \frac{1}{D^2_{ab}(\mathbf{A})} D_i^{bc}(\mathbf{A})\dot{A}_i^c \ , \tag{48}$$

the electric field can be written as

$$E_i^a = \Pi_{ij}^{ab}(\mathbf{A})\dot{A}_j^b \ , \tag{49}$$

with the projection operator

$$\Pi_{ij}^{ab}(\mathbf{A}) = \delta_{ij}\delta^{ab} - D_i^{ac}(\mathbf{A})\frac{1}{D^2_{cd}(\mathbf{A})}D_j^{db}(\mathbf{A}) \ . \tag{50}$$

We assume that zero modes of the differential operator $D^2_{ab}(A)$ are absent. Consideration of zero modes is under current investigation [23]. In the case of $A = 0$, this projection operator reduces to the transverse one $\Pi_{ij}^{ab}(\mathbf{A}=0) = \delta^{ab}\delta_{ij}^T \equiv \delta^{ab}\left(\delta_{ij} - \partial_i\partial_j/\partial^2\right)$.

The gauge invariant reduced Lagrangian can be written as

$$\mathcal{L}^{Red}(\mathbf{A}) = \frac{1}{2}\left[\left(\Pi^{ab}_{ij}(\mathbf{A})\dot{A}^b_j\right)^2 - B^{a\,2}_i(\mathbf{A})\right] \; . \tag{51}$$

Note that we still have

$$\frac{\delta\mathcal{L}^{Red}}{\delta\dot{A}^a_i} = E^a_i \;, \quad i = 1,2,3 \; . \tag{52}$$

Due to the property $\Pi^2 = \Pi$ of the projection operator the gauge invariant reduced Hamiltonian can be written in the form

$$H^{Red}(\mathbf{A},\mathbf{E}) = \int d^3x \frac{1}{2}\left[E^a_i \Pi^{ab}_{ij}(\mathbf{A})E^b_j + B^{a\,2}_i(\mathbf{A})\right] \; . \tag{53}$$

The non-Abelian projection operator has been inserted between the cartesian electric fields E_i, which as variables of the Hamiltonian lost their transversality property. The above form (53) is only one of many possible forms $E\Pi E$, $(\Pi E)^2$, $(\Pi E)\Pi(\Pi E)$,... Whereas in QED they are equivalent due to the property $\Pi^2 = \Pi$ and the possibility to perform partial integrations, in QCD they are inequivalent due to the presence of the A field in the covariant derivatives and lead to different operator orderings of E and A after quantization. The simplest choice $E\Pi E$ in (53) will be correct at least for our investigation of a squeezed homogeneous condensate, as discussed in the next paragraph. Although the form (53) is gauge invariant classically, we did not yet succeed in showing explicitly, that the corresponding Hamilton operator satisfies (43).

Thus in our treatment the role of gauge fixing is played by the projection operator (50), for deatils see [20,21,23]. The nonabelian chromomagnetic field projects onto the generalized transverse component of the A field quite analogous to the form $E^a_i \Pi^{ab}_{ij}(A)E^b_j$ for the chromoelectric field.

C. Spectrum of quasigluon excitations

We shall consider the squeezed vacuum containing a colourless homogeneous condensate of gluon pairs for the Hamiltonian

$$: H(\mathbf{A},\mathbf{E}) := \int \mathrm{d}^3x \; \frac{1}{2}\left[: E^a_i \Pi^{ab}_{ij}(\mathbf{A})E^b_j : + : B^{a\,2}_i(\mathbf{A}) :\right] \; . \tag{54}$$

We have introduced normal ordering with respect to creation a^+_p and annihilation operators a_p defined with respect to some open $\tilde{\omega}(p)$ in close analogy to our definitions (5) and (8) introduced in Section II for the $\lambda\varphi^4$ model. A four-gluon interaction term occurs in the $AAAA$ term of the magnetic part $B^{a\,2}_i(\mathbf{A})$ and in the kinetic term $E^a_i \Pi^{ab}_{ij} E^b_j$. In analogy to the $\lambda\varphi^4$ model we perform Wick reordering to the new squeezed vacuum and consider a homogeneous and colourless condensate ($f_{p\neq 0} = 0, f_0 \neq 0$) with the contraction

$$< 0 \,|\, A^a_i(p_1)A^b_j(p_2) \,|\, 0 > \; = < 0_B \,|\, (A^B)^a_i(p_1)(A^B)^b_j(p_2) \,|\, 0_B >= \delta_{ij}\delta^{ab}\delta_{p_1,0}\delta_{p_2,0}C \;\;, \tag{55}$$

where $A(\mathbf{p}), E(\mathbf{p})$ are the Fourier transforms of $A(\mathbf{x}), E(\mathbf{x})$ in analogy to (16), and the reordering formula for zero momentum gluon fields

$$: A^a_k(p=0)A^b_l(p=0) :=:: (A^B)^a_k(p=0)(A^B)^b_l(p=0) :: \mathrm{e}^{2f_0} + \tilde{C}\delta^{ab}\delta_{k,l} \;, \tag{56}$$

is in analogy to Eq. (16), where $\tilde{C} = C\left(\mathrm{e}^{2f_0} - 1\right)$. The corresponding reordering formula for the quartic term is calculated in Appendix C.

The Wick reordering of the magnetic part of the Hamiltonian (B6),

$$\frac{1}{2}\int \mathrm{d}^3x \;\; : B^{a\,2}_i(\mathbf{A}) : \; = \frac{1}{2}\int \mathrm{d}^3x \;\; \left\{: (\partial_j A^a_k)\delta^T_{kl}(\partial_j A^a_l) : + 2gf^{abc} : (\partial_j A^a_k)A^b_j A^c_k : + \frac{1}{2}g^2 f^{abc}f^{ade} : A^b_j A^c_k A^d_j A^e_k :\right\} \;, \tag{57}$$

leads to the result (see Appendix C)

$$\begin{aligned}\frac{1}{2}\int \mathrm{d}^3x \;\; : B^{a\,2}_i(\mathbf{A}) : \; &= g^2\frac{3}{2}\frac{N_c}{V}(N_c^2-1)(\tilde{C})^2 + \frac{1}{2}\sum_{p\neq 0}\left[(p^2 + 2g^2 N_c\tilde{C}/V)\delta_{ij} - p_i p_j\right] : A^a_i(p)A^a_j(-p) : \\ &+ \frac{1}{4V}g^2 \sum_{p_1\ldots p_4\neq 0} f^{abc}f^{ade} : A^b_j(p_1)A^c_k(p_2)A^d_j(p_3)A^e_k(p_4) : \delta_{p_1+p_2+p_3+p_4,0} \; .\end{aligned} \tag{58}$$

The first term corresponds to the conventional definition of the gluon condensate [5]

$$G^2 = \frac{g^2}{\pi^2} < \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a} >= \frac{g^2}{2\pi^2} < B_i^2 > \\ = \frac{N_c^2 - 1}{2\pi^2 3N_c} (3N_c g^2 \tilde{C}/V)^2 \ . \tag{59}$$

The second term in Eq.(58) includes the mass of the quasigluons which have both transverse and longitudinal parts, as the operation of the reordering destroys the projection properties of the non-Abelian magnetic field. This fact can be understood as spontaneous gauge symmetry breaking, and is the Bose-analogy of the spontaneous chiral symmetry breaking for the constituent quarks [24–26], which is also realized by the corresponding Bogoliubov transformation of the "squeezed" type. It is well known that a massive vector field requires for the description a number of degrees of freedom which exceeds that provided after fixation of a gauge in QCD. Note that the fixing of a gauge results in an elimination of the longitudinal components of the vector field and is inconsistent with the concept of a mass [14]. So, the method of projection onto gauge invariant variables [19] used here is more adequate to the phenomenon of spontaneous gauge symmetry breaking than the conventional gauge fixing method. Similarly to the above magnetic energy the projector Π^{ab}_{ij} in the electric energy $E^a_i \Pi^{ab}_{ij}(\mathbf{A}) E^b_j$ in (54) is destroyed by the Wick reordering procedure leading to the kinetic energy contribution

$$: E^a_i E^b_j :< \Pi^{ab}_{ij}(\mathbf{A}) > \ . \tag{60}$$

The expression $< \Pi^{ab}_{ij}(\mathbf{A}) >$ can be determined in the low energy limit $p^2 \sim 0$:

$$< \Pi^{ab}_{ij}(\mathbf{A}) > \Big|_{p\sim 0} \simeq \delta_{ij}\delta^{ab} - < f^{agc} A^g_i \frac{1}{\sum_k f^{cme} A^m_k f^{end} A^n_k} f^{dlb} A^l_j >= \frac{2}{3} \delta_{ij} \delta^{ab} \ . \tag{61}$$

which can easily be checked by summation over colour and space indices. The Eqs. (58), (60) and (61) allow us to find the effective Hamiltonian for the quasigluon excitations in the low-energy limit:

$$H_{\rm eff}(\mathbf{E}, \mathbf{A}) = \frac{1}{2} (\frac{2}{3}) \int \mathrm{d}^3 x (E_i^2 + m_g^2 A_i^2) \ , \tag{62}$$

with the quasigluon mass

$$m_g^2 = 3 N_c g^2 \frac{\tilde{C}}{V} \ . \tag{63}$$

The corresponding effective low energy Lagrangian is

$$\mathcal{L}_{\rm eff}(\mathbf{A}) = \frac{1}{2} (\frac{2}{3}) \int \mathrm{d}^3 x (\dot{A}_i^2 - -m_g^2 A_i^2) \ . \tag{64}$$

The quasigluon mass m_g in the low energy limit is determined by the vacuum expectation value using the relations (59) and (63):

$$m_g = \sqrt{\frac{3\pi G}{2}} \ . \tag{65}$$

We have shown that the Bogoliubov condensation of gluon pairs leads to a nonvanishing contraction $\tilde{C}$ of gluon fields which results in the spontaneous gauge symmetry breaking and the occurence of a gluon mass. We have obtained a new gluon mass formula for the low energy limit of QCD.

In the following Section we examine consequences of the present approach to the low energy sector of QCD for the η' mass formula.

IV. APPLICATIONS: QUASIGLUON MASS AND $\eta - \eta'$ MASS DIFFERENCE

According to the Bogoliubov condensate approach, the contraction $\tilde{C}$ is a phenomenological parameter of the "squeezed" vacuum state and is directly related to the macroscopic occupation of the zero momentum quasigluon

state and therefore to the gluon mass. We suggest to use this relationship for the fixation of the squared mass difference

$$m_{\eta'}^2 - m_\eta^2 = \Delta m_{\eta'}^2 = 0.616 \text{ GeV}^2. \tag{66}$$

According to conventional approaches to the determination of the η' mass [32], we suppose that the mass difference (66) is determined by the $\eta' \to \eta'$ transition through the process of the anomalous decay of the η' meson into the gluon condensate B_i^a and a collective gluon excitation E_i^a. The effective Lagrangian of such a process can be derived according to Ref. [32], see also [33], with the result

$$\mathcal{L}_{\eta'} = \frac{1}{4} F_{\mu\nu}^a \tilde{F}^{a\mu\nu} \ \eta' c_\pi \ , \tag{67}$$

where $c_\pi = \sqrt{3}\alpha_s/(\pi F_\pi)$, $\alpha_s = g^2/4\pi$ and $F_\pi = 93$ MeV.

For the derivation of an effective Hamiltonian for this process, we use the sum of the Lagrangians (67) and (64),

$$\mathcal{L}_{\text{eff}}(A) = \frac{1}{2}(\dot{A}_i)^2 - c_\pi \eta' \dot{A}_i B_i + \dots \ . \tag{68}$$

From (68) follows

$$E_i = \dot{A}_i - c_\pi \eta' B_i \ , \tag{69}$$

such that the effective Hamiltonian reads

$$\mathcal{H}_{\text{eff}} = E_i \dot{A}_i - \mathcal{L}_{\text{eff}}(A) \tag{70}$$

$$= \frac{1}{2}(E_i + c_\pi \eta' B_i)^2 + \dots \ . \tag{71}$$

The effective Hamiltonian of the $\eta' \to \eta'$ transition has the form

$$\mathcal{H}_{\eta' \to \eta'} = \frac{1}{2}(\eta')^2 (B_i^a(b))^2 c_\pi^2 \ , \tag{72}$$

where b is the constant part of the A field.

Thus, we have

$$\Delta m_{\eta'}^2 = (\frac{3\alpha_s^2}{\pi^2 F_\pi^2}) \ < (B_i^a(b))^2 > . \tag{73}$$

This equation together with Eqs.(59) and (65) leads to a relation between the quasigluon and η' masses:

$$\Delta m_{\eta'}^2 = \frac{2}{3} \frac{\alpha_s}{\pi^3 F_\pi^2} \ m_g^4 \ . \tag{74}$$

Choosing in the low-energy region $\alpha_s \simeq 1$ we can estimate ¿from this formula the value of the quasigluon mass and by formula (65) then the corresponding value of the gluon condensate as

$$m_g = 0.71 \text{ GeV} \quad , \qquad G^2 = 0.011 \text{ GeV}^4 \ , \tag{75}$$

which is in the agreement with earlier estimates [10,5]. Note that the process of the decay of η' into gluon fields by means of the Hamiltonian (67) is forbidden, as the vacuum expectation value from the magnetic field $< 0_B | B_i^a(b) | 0_B >$ is equal to zero.

V. CONCLUSIONS

In the present paper we have considered the consequences of a squeezed vacuum for the single particle excitation spectrum in the gluon sector of QCD by applying the concept of the Bogoliubov theory of superfluidity to field theory. We have considered a squeezed homogeneous colourless condensate of zero momentum gluon pairs. The

macroscopic occupation (squeezing) of the zero momentum mode has been achieved through Wick reordering of the QCD Hamiltonian and is characterized by a parameter $\tilde{C}$ which describes the magnitude of the condensate.

The presence of the condensate leads to the occurrence of a gluon mass and thus to spontaneous gauge symmetry breaking, i.e. the gauge invariance of the Hamiltonian is not shared by the squeezed vacuum. Instead of eliminating unphysical degrees of freedom by fixing a gauge we use a projection operator method resulting from the formal solution of Gauss law. We show that the occurence of a condensate leads to a destruction of the projection property so that the generation of a mass is accompanied by the appearance of the necessary longitudinal component for the gauge field. We found that the quasigluon spectrum depends on the parameter $\tilde{C}$ of the squeezed representation, which yielded a relation between the quasigluon mass and the gluon condensate. We have fixed the quasigluon mass from the squared mass difference $m^2_{\eta'} - m^2_\eta = 0.616$ GeV^2 of η and η' and found that the corresponding value of the gluon condensate $G^2 = 0.012\mathrm{GeV}^4$ then agrees well with the standard value $G^2 = 0.01\mathrm{GeV}^4$ obtained by Shifman, Vainshtein and Zakharov.

In this paper we have populated the zero momentum state directly with quasigluons whose dispersion relation was then determined selfconsistently by demanding diagonality of the one-particle sector of the Hamiltonian for nonzero momentum. This should be considered as a first attempt to explain the concept of the squeezed vacuum. In a more rigorous treatment the zero momentum state should first be occupied macroscopically with massless gluons using the freedom due to the infrared singularity of massless theories, and the resulting one-particle sector of the Hamiltonian should then be diagonalized by transformation to quasiparticles. This has been carried out for the much simpler $\lambda\phi^4$ theory in a separate work [18] which shows how in a more rigorous treatment the renormalization of both the mass and the bare coupling are included.

Important extensions of the present approach include the study of small deviations from a homogeneous condensate, the inclusion of quark degrees of freedom and the generalization to finite temperatures. These issues are currently under investigation and will be reported in a forthcoming paper.

ACKNOWLEDGEMENT

We are grateful to D. Ebert, A.V. Efremov, E.A. Kuraev, H. Leutwyler, L.N. Lipatov, D.V. Shirkov, A.A. Slavnov, O.I. Zav'ialov for fruitful discussions. We thank S. Schmidt and D. Rischke for useful comments and critical reading of the manuscript. The work was supported in part by the RFFI, Grant No. 95-02-14411 and the Federal Minister for Research and Technology (BMFT) within Heisenberg-Landau Programme. Two of us (V.N.P. and M.K.V.) acknowledge the financial support provided by the Max-Planck-Gesellschaft and the hospitality of the MPG Arbeitsgruppe "Theoretische Vielteilchenphysik" at the Rostock University, where part of this work has been done. One of the authors (V.N.P.) acknowledges the hospitality of the International Centre for Theoretical Physics in Trieste.

APPENDIX A: THE WEAKLY NONIDEAL BOSE GAS MODEL

The Bogoliubov theory of the weakly interacting Bose gas [7] is described by the nonrelativistic Hamiltonian

$$H = \sum_p \frac{p^2}{2m} a^+_p a_p + \frac{U_0}{2V} \sum_{p_1 p_2 p'_1 p'_2} a^+_{p_1} a^+_{p_2} a_{p'_2} a_{p'_1} \delta_{p_1+p_2, p'_1+p'_2} \ . \tag{A1}$$

The operators a^+_p, a_p are the creation and annihilation operators of bosons in the state p satisfy the commutation relations

$$[a_p, a^+_{p'}] = \delta_{pp'}, \tag{A2}$$

where p stands for momentum and internal quantum numbers. The coupling constant U_0 is defined by the scattering amplitude of slow particles, V is the volume of the system. We figure out the original work of Bogoliubov [7], for a more recent presentation of the theory of the weakly interacting Bose gas see [34], [35].

In the Bogoliubov derivation of the superfluid spectrum one can distinguish three points:

1. A macroscopic occupation of the zero momentum state ($p = 0$) is assumed so that in the thermodynamic limit a finite density of the condensate

$$n_B = \mathrm{lim}_{\mathrm{th}} \frac{N_0}{V} \neq 0 \tag{A3}$$

occurs, where N_0 denotes the number of particles in the condensate. Therefore, the operators a_0^+, a_0 in the thermodynamic limit (A3) are described as c-numbers

$$a_0 \simeq a_0^+ \simeq \sqrt{N_0}. \tag{A4}$$

This description, strictly speaking, should be completed by defining a representation for the condensate which in the present work is given below.

2. The next step is the expansion of the Hamiltonian (A1) around these c-numbers

$$\sum a^+_{p_1} a^+_{p_2} a_{p_2'} a_{p_1'} = N_0^2 + N_0 \sum_{p\neq 0} (a_p^+ a_{-p}^+ + a_p a_{-p} + 4 a_p^+ a_p) + O[a^3_{p\neq 0}] \ . \tag{A5}$$

Taking into account the conservation of the total number of particles N and rewriting

$$N_0 = N - \sum_{p\neq 0} a_p^+ a_p \quad ; \left(\frac{N - N_0}{N} \ll 1\right),$$

in Eq. (A5) and neglecting terms of higher than second order in the particle operators $a_{p\neq 0}, a^+_{p\neq 0}$, the Hamiltonian (A1) transforms into

$$H = \frac{N}{2}\nu + \sum_{p\neq 0} \left[a_p^+ a_p \varepsilon_p + \frac{\nu}{2}\left(a_p^+ a_{-p}^+ + a_p a_{-p}\right)\right] + O[a^3_{p\neq 0}], \tag{A6}$$

where

$$\varepsilon_p = \frac{p^2}{2m} + \nu, \quad \nu = U_0 \frac{N}{V}. \tag{A7}$$

3. The last step is the diagonalization of (A6) using the Bogoliubov transformation, i.e. the transition to the operators of quasiparticles b_p^+ and b_p for $p \neq 0$

$$\begin{aligned} b_p &= U^{-1} a_p U = \cosh(f_p) a_p + \sinh(f_p) a^+_{-p}, \\ b_p^+ &= U^{-1} a_p^+ U = \cosh(f_p) a_p^+ + \sinh(f_p) a_{-p}, \end{aligned} \tag{A8}$$

where

$$U = \exp\left\{\sum_p \frac{f_p}{2}(a_p^+ a_{-p}^+ \ - \ a_p a_{-p})\right\} \ . \tag{A9}$$

The b_p satisfy the same commutation relations as the a_p. The function f_p is found from the requirement of the disappearance of nondiagonal terms as

$$f_p = \frac{1}{2}\mathrm{arth}\left[\frac{\nu}{\varepsilon_p}\right] \ , \tag{A10}$$

so that the Hamiltonian (A6) gets the form

$$H = \frac{N}{2}\nu - \frac{1}{2}\sum_{p\neq 0}\left(\varepsilon_p - \omega_B(p)\right) + \ \sum_{p\neq 0} b_p^+ b_p \ \omega_B(p) + \ O[b^3_{p\neq 0}] \ , \tag{A11}$$

where ω_B is the spectrum of excitations in a superfluid liquid

$$\omega_B^2(p) = \varepsilon_p^2 - \nu^2 = \left(\frac{p^2}{2m}\right)^2 + \frac{p^2}{2m}\left(2U_0 \ \frac{N}{V}\right) \ , \tag{A12}$$

which is determined by the condensate density $n_B = N_0/V \cong N/V$ and by the coupling constant U_0.

In the low momentum region this expression describes the Landau sound and the particle excitations with energy $\big(p^2/2m\big)$ disappear.

Note that the vacuum energy E_0 contains a divergent sum which can be renormalized by expressing it in terms of the physical scattering length a instead of the bare coupling U_0 (see [34], p. 318).

In his paper [7], Bogoliubov did not determine the representation of the condensate state for which Eq. (A4) is fulfilled. Usually one assumes that of the coherent state

$$|0_{\rm C}> \;\; = \exp\left\{\sum_p c_0(a_0^+ + a_0)\right\} \; |0> \;\; , \; c_0 = \sqrt{N_0} \; , \tag{A13}$$

for which holds

$$< 0_{\rm C}|a_0|0_{\rm C}> \;\; = \;\; < 0_{\rm C}|a_0^+|0_{\rm C}> \;\; = \sqrt{N_0} \; , \tag{A14}$$

corresponding to Eq. (A4).

However, to get the Bogoliubov result it is enough to assume the weaker condition

$$(a_0^+)^2 \simeq a_0^2 \simeq a_0^+ \, a_0 \sim N_0 \; , \tag{A15}$$

rather than (A4). These relations are fulfilled for the representation of the condensate state which is given by the same Bogoliubov transformation as for $p \neq 0$ (A9):

$$|\, 0_B >= U_B^{-1} \;\; |\, 0> \;\; , \tag{A16}$$

where

$$U_{B_0} = \exp\left\{\frac{f_0}{2}(a_0^+ a_{-0}^+ \; - \; a_0 a_{-0})\right\} \; . \tag{A17}$$

The inverse of the unitary operator (A9) defines also the transformation of the old into a new vacuum state for momenta $p \neq 0$. In quantum optics the vacuum $|\, 0_B >$ is called 'squeezed vacuum', see e.g. [27].

For the "squeezed" vacuum representation (A16) of the condensate we have the realization (A15)

$$\begin{aligned} < 0_B \,|\, a_0^2 \,|\, 0_B > \; &= < 0_B \,|\, (a_0^+)^2 \,|\, 0_B > \;\; = -\cosh f_0 \, \sinh f_0 \;\; , \\ < 0_B \,|\, a_0^+ a_0 \,|\, 0_B > \; &= (\sinh f_0)^2 = N_0 \; , \end{aligned} \tag{A18}$$

and at large N_0 (A15) means that

$$\begin{aligned} &- -\cosh f_0 \, \sinh f_0 \simeq (\sinh f_0)^2 \simeq N_0 \to \infty \;\; , \\ &f_0 \sim -\frac{1}{2} \ln 4N_0 \; . \end{aligned} \tag{A19}$$

The choice of the squeezed vacuum is more prefable from the point view of a general consideration of all momenta, $p = 0$ *and* $p \neq 0$. Together, the Bogoliubov transformation now is given by the product UU_B.

APPENDIX B: GAUGE INVARIANT VARIABLES

The unphysical components of the gluon fields are formally eliminated by the transformation to gauge invariant variables [19,20] which are functionals constructed using the solution (48)

$$A_i^I[\mathbf{A}] \equiv V(\mathbf{A})(A_i + \partial_i)V(\mathbf{A})^{-1} \; . \tag{B1}$$

The matrix V is defined from the equation

$$V(A_0[\mathbf{A}] + \partial_0)V^{-1} = 0 \Rightarrow V(\mathbf{A}) = T\exp(\int^t A_0[\mathbf{A}]dt^{'}) \tag{B2}$$

(up to a stationary matrix as the time boundary condition). The invariance of these functionals under arbitrary time dependent gauge transformations $v(\mathbf{x}, t)$

$$A_i^I[\mathbf{A}^v] = V(\mathbf{A})v^{-1}v(A_i + \partial_i)v\ v^{-1}V(\mathbf{A}) = A_i^I[\mathbf{A}]\ , \tag{B3}$$

follows from the transformation properties of A_i in (32) and of $V(\mathbf{A})$:

$$V(\mathbf{A}^v) = V(\mathbf{A})v^{-1}\ . \tag{B4}$$

which follows from (B2) and (32). As consequence of this the variables (B1) represent only 2 $(N_c^2 - 1)$ independent degrees of freedom. They contain hidden projection operators onto generalized transverse components similar to the magnetic field which satisfies the Bianchi identity (37). The projection operator is contained (different to the QED case) not in the A_i^I themselves, but only in their time derivatives $\dot{A}_i^I$ which satisfy the "Bianchi type" identities

$$D_i^{ab}(\mathbf{A}^I)\dot{A}_i^{I^b} \equiv 0\ . \tag{B5}$$

In the terms of the functionals (B1) the Lagrangian (29) takes the form

$$\mathcal{L}^{Red}(\mathbf{A}^I) = \frac{1}{2}\ \left[\dot{A}_i^{I^a 2} - -B_i^{a2}(\mathbf{A}^I)\right]\ . \tag{B6}$$

The canonical momenta to the spatial fields A_i^{Ia} are

$$E_i^{I^a} \equiv \frac{\delta\mathcal{L}}{\delta\dot{A}_i^{I^a}} = \dot{A}_i^{I^a}\ . \tag{B7}$$

The Hamiltonian becomes

$$H^{Red}(\mathbf{A}^I, \mathbf{E}^I) = \int d^3x \frac{1}{2}\left[E_i^{Ia2} + B_i^a(\mathbf{A}^I)^2\right]\ . \tag{B8}$$

It follows from (B5) that the electric fields ${E^I}_i^a$ satisfy the Gauss constraint

$$D_i^{ab}(\mathbf{A}^I){E^I}_i^b = 0\ . \tag{B9}$$

Like the Bianci identity (37) for the magnetic field, this shows the generalized transversality of the invariant electric fields ${E^I}_i^a$.

In order to quantize the theory one could then like in QED impose the following canonical commutation relations on the physical variables A^I and E^I:

$$[E_i^{Ia}(\mathbf{x}), A_j^{Ib}(\mathbf{x}')] = i\delta^{ab}\delta_{ij}\delta(\mathbf{x} - \mathbf{x}')\ . \tag{B10}$$

In QCD, however, this leads to a contradiction when applying the covariant derivative on it.

Instead one has to impose canonical commutation relations directly on the three cartesian fields A_i and E_i and write both E^I and A^I as functionals of E and A. Whereas the form of $A^I[\mathbf{A}]$ is known by construction (B1), the functional form of $E^I[\mathbf{E}, \mathbf{A}]$ in terms of E and A has not been found yet, but is subject of intensive research [36]. However, even if this problem is solved there remains still the question of correct ordering of the operators A and E.

APPENDIX C: WICK REORDERING OF GLUON FIELDS

In this appendix we perform the Wick reordering of the magnetic part of the Hamiltonian (B6),

$$\frac{1}{2}\int \mathrm{d}^3x\ \ : B_i^{a2}(\mathbf{A}) :\ = \frac{1}{2}\int \mathrm{d}^3x\ \left\{: (\partial_j A_k^a)\delta_{kl}^T(\partial_j A_l^a) : +2gf^{abc} : (\partial_j A_k^a)A_j^b A_k^c : +\frac{1}{2}g^2 f^{abc}f^{ade} : A_j^b A_k^c A_j^d A_k^e :\right\}\ , \tag{C1}$$

which reads in momentum space

$$\begin{aligned}\frac{1}{2}\int \mathrm{d}^3x\ \ : B_i^{a2}(\mathbf{A}) :\ = \frac{1}{2}\Big\{ &\sum_p : A_k^a(p)\left[p_k p_l - p^2\delta_{kl}\right]A_l^a(-p) : \\ &+2g^2\frac{1}{\sqrt{V}}f^{abc}\sum_{p_1 p_2} i(p_1+p_2)_j : A_k^a(p_1+p_2)A_j^b(-p_1)A_k^c(-p_2) : \\ &+\frac{1}{2}g^2\frac{1}{V}f^{abc}f^{ade}\sum_{p_1\ldots p_4} : A_j^b(p_1)A_k^c(p_2)A_j^d(p_3)A_k^e(p_4) : \delta_{p_1+p_2+p_3+p_4,0}\Big\}\ .\end{aligned} \tag{C2}$$

The reodering of the first two terms on the r.h.s. of Eq. (C2) gives no extra contraction contribution. For the Wick reordering of the third term we write

$$
\begin{aligned}
f^{abc} f^{ade} : A_j^b(p_1) A_k^c(p_2) A_j^d(p_3) A_k^e(p_4) : \; = \; & f^{abc} f^{ade} A_j^b(p_1) A_k^c(p_2) A_j^d(p_3) A_k^e(p_4) \\
& - f^{abc} f^{ade} < A_j^b(p_1) A_j^d(p_3) > A_k^c(p_2) A_k^e(p_4) \\
& - - f^{abc} f^{ade} A_j^b(p_1) A_j^d(p_3) < A_k^c(p_2) A_k^e(p_4) > \\
& - f^{abc} f^{ade} < A_j^b(p_1) A_k^c(p_2) > A_j^d(p_3) A_k^e(p_4) \\
& - - f^{abc} f^{ade} A_j^b(p_1) A_k^e(p_4) < A_k^c(p_2) A_j^d(p_3) > \\
& + f^{abc} f^{ade} < A_j^b(p_1) A_j^d(p_3) >< A_k^c(p_2) A_k^e(p_4) > \\
& + f^{abc} f^{ade} < A_j^b(p_1) A_k^c(p_2) >< A_j^d(p_3) A_k^e(p_4) > \ . \qquad \text{(C3)}
\end{aligned}
$$

Using formula (55) and the identities

$$
\begin{aligned}
\sum_{abc} f^{abc} \ f^{abc} &= N_c(N_c^2 - 1) \ , \\
\sum_{ab} f^{abc} \ f^{abe} &= N_c \delta_{ce} \ ,
\end{aligned}
$$

in Eq. (C3) we thus have

$$
\begin{aligned}
\sum_{p_1 \ldots p_4} f^{abc} f^{ade} \ : \ & A_j^b(p_1) A_k^c(p_2) A_j^d(p_3) A_k^e(p_4) : \delta_{p_1+p_2+p_3+p_4,0} = \\
& 6N_c(N_c^2 - 1)C^2 - -4N_c C \sum_p A_i^a(p) A_i^a(-p) \\
& + \sum_{p_1 \ldots p_4} f^{abc} f^{ade} A_j^b(p_1) A_k^c(p_2) A_j^d(p_3) A_k^e(p_4) \delta_{p_1+p_2+p_3+p_4,0} \\
= \ & 6N_c(N_c^2 - 1)\left(C^B\right)^2 + 4N_c C^B \sum_p :: A_i^a(p) A_i^a(-p) :: \mathrm{e}^{f_p + f_{-p}} \\
& + \sum_{p_1 \ldots p_4} f^{abc} f^{ade} :: A_j^b(p_1) A_k^c(p_2) A_j^d(p_3) A_k^e(p_4) :: \mathrm{e}^{f_{p_1} + f_{p_2} + f_{p_3} + f_{p_4}} \delta_{p_1+p_2+p_3+p_4,0} \ . \qquad \text{(C4)}
\end{aligned}
$$

Putting all together,

$$
\begin{aligned}
\frac{1}{2} \int \mathrm{d}^3 x \ : B_i^{a\,2}(\mathbf{A}) : \ = \ & g^2 \frac{3}{2} \frac{N_c}{V} (N_c^2 - 1)\left(C^B\right)^2 + \frac{1}{2} \sum_p \left[(p^2 + 2g^2 N_c C^B / V)\delta_{ij} - p_i p_j \right] :: A_i^a(p) A_j^a(-p) :: \mathrm{e}^{f_p + f_{-p}} \\
& + \frac{1}{4V} g^2 \sum_{p_1 \ldots p_4} f^{abc} f^{ade} :: A_j^b(p_1) A_k^c(p_2) A_j^d(p_3) A_k^e(p_4) :: \mathrm{e}^{f_{p_1} + f_{p_2} + f_{p_3} + f_{p_4}} \delta_{p_1+p_2+p_3+p_4,0} \ . \qquad \text{(C5)}
\end{aligned}
$$

This proves the result of Eq. (58) used in the main text.

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